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Report for the PhD thesis by Aleksandra Krawiec:
Discrimination of quantum measurements

Quantum information is now an established topic in physics and lies at the heart of the so-called "second quantum revolution". This revolution is characterised by entanglement-enabled technologies that offer new perspectives in potential applications such as precision measurements, communication protocols, and quantum simulations. Furthermore, studying physical systems as carriers of information enables us to gain deeper insights into the fundamental properties and features of the quantum world. As our understanding of the theory of quantum information processing grows, the recent perspective is to consider quantum transformations, rather than states, as carriers of information. This perspective is relevant in various applications, including channel discrimination, channel tomography, multi-round quantum games, and quantum networks.

Aleksandra Krawiec's PhD thesis focuses on the discrimination of quantum measurements, a subject which aligns with this new trend. In essence, a quantum measurement can be regarded as a channel with a quantum input (the state to be measured) and a classical output (the outcome of the measurement). From this perspective, the discrimination between two quantum measurement can be framed as a quantum channel discrimination. Consequently, the probability of correct discrimination between two measurements in the single shot scenario is related to the cb-norm distance between the corresponding quantum-to-classical channels. For the discrimination of von Neumann measurements, this distance can be related to the cb-norm distance of unitary channels. This is the content of Theorem 2, one of the main findings of the thesis, which is discussed in Chapter 3 and proved in Appendix A. This result reduces the optimal discrimination between von Neumann measurements to a minimisation problem over the compact set of diagonal unitary matrices. Finding an analytical solution for this minimisation problem is generally challenging. To address this, presenting the complete solution for the two-dimensional case, which is relatively straightforward to obtain, would be beneficial. In section 3.4 the author studies the single shot discrimination between two SIC-POVMs which differ by a re-labeling of the outcomes. This more constrained setting allows for quantitative upper and lower bounds of the cb-norm distance, which are presented in Proposition 4. This is surely an interesting result, but a bit academic. The discrimination of covariant POVMs (i.e. the discrimination between POVMs with different seed) is another analysis that could be interesting to explore.

When multiple uses of measurements are available, various discrimination strategies can be employed. For instance, one approach is to apply all N uses of the unknown measurement in parallel on a multipartite state. Alternatively, a sequential (or adaptive) strategy can be utilised, where the outcome of one measurement is used to adjust the input query for the next measurement. It is worth noting that the parallel strategy is a special case of the sequential strategy, where no information processing occurs between successive queries of the unknown measurement. While adaptive discrimination strategies have been shown to outperform parallel ones for quantum channel discrimination, the question arises whether the same holds true for the discrimination of quantum measurements with multiple uses. This problem is addressed in Chapter 4 of the thesis, and Theorem 6 demonstrates that the parallel scheme is optimal for the discrimination of von Neumann measurements. However, if we do not restrict ourselves to von Neumann measurements, there exist examples of POVMs for which the sequential strategy is strictly optimal. Section 4.5 showcases this characteristic with certain SIC-POVMs.

Another approach to discrimination is the so-called unambiguous one, which allows for an inconclusive answer. This scenario, both in the single-shot and multiple-use cases, is considered in Chapter 5 of the present thesis. Theorem 7 and Corollary 6 are the analogues of Theorem 2 for the unambiguous discrimination of von Neumann measurements. Similar to Theorem 2, the optimal solution is given in terms of a minimisation over the set of diagonal unitary matrices. Also in this case, it would be instructive to see the explicit solution for the qubit case. Regarding the multiple-shot unambiguous discrimination of von Neumann measurements, Theorem 9 shows that the parallel strategy is optimal.

Chapter 6 presents another approach to discrimination, known as hypothesis testing. In this scenario one differentiates between two types of error: false positive and false negative. Theorem 10 presents an interesting result regarding quantum channel certification, and this result is further specified for the certification of von Neumann measurements in Theorem 11.

In my opinion, Aleksandra Krawiec has prepared an excellent thesis which is well-written and contains a coherent corpus of original scientific results, which significantly advance our understanding of the mathematical structure of quantum theory. These results have been published in high-impact journals. The thesis clearly manifests Aleksandra Krawiec's proficiency and capability of promoting original research in the field of Quantum Information. The presented dissertation meets the statutory requirements for doctoral theses in the field of engineering and technology in the discipline information and communication technology.

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