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Descriptor continuous- and discrete-time linear systems with zero transfer matrices

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Abstract. In this paper, necessary and sufficient conditions for zeroing of the transfer matrices of descriptor continuous-time and discrete-time linear systems are established. The conditions are illustrated by simple numerical examples of the descriptor continuous-time and discrete-time linear systems. Also some remarks on the systems with delays in control are given.

Keywords: controllability and observability; descriptor continuous-time linear systems; descriptor discrete-time linear systems; zero transfer matrices.

1. INTRODUCTION

In the theory of linear control systems the notions of controllability, and observability introduced by Kalman [9, 10] play fundamental role [11–13]. Some recent developments on this crucial notions have been presented in the papers [6–8, 13] and the references therein. On the other hand, the descriptor (aka singular or implicit) systems have been subject to intensive investigations in recent years (see e.g., [1, 2, 5] for details). In this paper we shall concentrate on controllability, and its dual concept observability of descriptor continuous-time and discrete-time linear systems. Moreover, as direct consequences of these notions, in the paper necessary and sufficient conditions for the zeroing of the transfer matrices of descriptor continuous-time and discrete-time linear systems are introduced and proved. Zeroing problem has some direct consequences when considering the decoupling of coupled systems, one of the most interesting problems in system theory and control. The decoupling control strategies allow us to simplify the control itself and also the identification procedure of the parameters of complex control systems in the context of noninteracting control (see e.g., [14] for details). Zeroing problem for the transfer matrix of Roesser model of 2-D linear systems was discussed in the paper [4].

The paper is organized as follows: in Section 2 the controllability and in Section 3 the observability of the descriptor linear systems are analyzed. Necessary and sufficient conditions for zeroing of the transfer matrices of the descriptor continuous-time and discrete-time linear systems have been established and illustrated by simple numerical examples in Sections 4 and 6, respectively. In Section 7 zeroing of transfer function for linear, continuous-time, descriptor systems with delays in control are considered. Concluding remarks are given in Section 8.

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2. CONTROLLABILITY OF DESCRIPTOR CONTINUOUS-TIME LINEAR SYSTEMS

Let us consider the descriptor, finite-dimensional, linear continuous-time system:

$$\mathbb{E}\dot{x} = \mathbb{A}x + \mathbb{B}u, \tag{1a}$$

$$y = \mathbb{C}x, \tag{1b}$$

where $t \in [0, t_f]$, and $x = x(t) \in \mathbb{R}^n$, $u = u(t) \in \mathbb{R}^m$, $y = y(t) \in \mathbb{R}^p$ are the state, input and output vectors respectively, and $\mathbb{E}, \mathbb{A} \in \mathbb{R}^{n\times n}$, $\mathbb{B} \in \mathbb{R}^{n\times m}$, $\mathbb{C} \in \mathbb{R}^{p\times n}$ are constant matrices. It is assumed that:

$$\det[\mathbb{E}s - \mathbb{A}] \neq 0. \tag{2}$$

In this case the equation (1a) has a unique solution, given in [5].

Remark 1. Note that, if

$$y = \mathbb{C}x + \mathbb{D}u, \quad \mathbb{D} \in \mathbb{R}^{p\times m} \tag{3}$$

then defining

$$\bar{y} = y - \mathbb{D}u = \mathbb{C}x \tag{4}$$

we may reduce the case (3) to (1b).

Definition 1 [3]**.** The system (1a) is called completely controllable if for any initial state $x(0) \in \mathbb{R}^n$ and every finite state $x_f \in \mathbb{R}^n$ there exists an input $u(t) \in \mathbb{R}^m$, for $t \in [0, t_f]$ such that $x(t_f) = x_f$.

Theorem 1. The system (1a) is completely controllable if and only if:

$$\operatorname{rank}[\mathbb{E}s - \mathbb{A}, \mathbb{B}] = n \quad \text{for all} \quad s \in \mathrm{C}, \tag{5a}$$

$$\operatorname{rank}[\mathbb{E}, \mathbb{B}] = n, \tag{5b}$$

where C is the field of complex numbers.

Proof of this theorem is given in [3]. □

The transfer matrix of the system (1) has the form:

$$\mathbf{T}(s) = \mathbb{C}[\mathbb{E}s - \mathbb{A}]^{-1}\mathbb{B}. \tag{6}$$

The transfer matrix (6) represents the controllable part of the system (1) [3].

3. OBSERVABILITY OF DESCRIPTOR CONTINUOUS-TIME LINEAR SYSTEMS

Let us consider the descriptor continuous-time linear system (1) satisfying the condition (2).

Definition 2 [3]**.** The system (1) is called completely observable if there exists $t_f > 0$ such that knowing the input $u(t)$, and the output $y(t)$ for $t \in [0, t_f]$ it is possible to find (compute) the initial state vector x_0 of the system.

Theorem 2. The system (1a) and (1b) is observable if and only if

$$\operatorname{rank}\begin{bmatrix}\mathbb{E}s - \mathbb{A}\\ \mathbb{C}\end{bmatrix} = n \quad \text{for all} \quad s \in \mathrm{C}, \tag{7a}$$

$$\operatorname{rank}\begin{bmatrix}\mathbb{E}\\ \mathbb{C}\end{bmatrix} = n. \tag{7b}$$

Proof of this theorem is given in [3]. □

Therefore, the transfer matrix (6) represents only the controllable and observable part of the system (1) [3].

Example 1. Let us consider the descriptor system (1a, 1b) with the matrices:

$$\mathbb{E} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix},$$

$$\mathbb{A} = \begin{bmatrix}1 & 1 & 2\\ 0 & 2 & -1\\ 0 & 0 & 1\end{bmatrix}, \quad \mathbb{B} = \begin{bmatrix}1 & 0\\ 0 & -1\\ 0 & 0\end{bmatrix}, \quad \mathbb{C} = \begin{bmatrix}0 & 1 & 0\end{bmatrix}. \tag{8}$$

This system satisfies the condition (2) since

$$\det[\mathbb{E}s - \mathbb{A}] = \begin{vmatrix}-1 & s-1 & -2\\ 0 & -2 & s+1\\ 0 & 0 & -1\end{vmatrix} = -2 \neq 0. \tag{9}$$

The system with matrices (8) satisfies the condition (5a)

$$\operatorname{rank}[\mathbb{E}s - \mathbb{A}, \mathbb{B}] = \operatorname{rank}\begin{bmatrix}-1 & s-1 & -2 & 1 & 0\\ 0 & -2 & s+1 & 0 & -1\\ 0 & 0 & -1 & 0 & 0\end{bmatrix}$$
$$= 3 = n. \tag{10}$$

but the condition (5b) is not satisfied

$$\operatorname{rank}[\mathbb{E}, \mathbb{B}] = \operatorname{rank}\begin{bmatrix}0 & 1 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & -1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix} = 2 < n = 3. \tag{11}$$

Therefore the system is not controllable. The system is also not observable. It satisfies the condition

$$\operatorname{rank}\begin{bmatrix}\mathbb{E}s - \mathbb{A}\\ \mathbb{C}\end{bmatrix} = \operatorname{rank}\begin{bmatrix}-1 & s-1 & -2\\ 0 & -2 & s+1\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix} = 3 = n. \tag{12}$$

but the condition (7b) is not satisfied

$$\operatorname{rank}\begin{bmatrix}\mathbb{E}\\ \mathbb{C}\end{bmatrix} = \operatorname{rank}\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 1 & 0\end{bmatrix} = 2 < n = 3. \tag{13}$$

The transfer matrix of the system has the form

$$\mathbf{T}(s) = \mathbb{C}[\mathbb{E}s - \mathbb{A}]^{-1}\mathbb{B} = \begin{bmatrix}0 & 1 & 0\end{bmatrix}$$
$$\times \begin{bmatrix}-1 & s-1 & -2\\ 0 & -2 & s+1\\ 0 & 0 & -1\end{bmatrix}^{-1} \begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 0.5\end{bmatrix}. \tag{14}$$

Let us note that in this case

$$\mathbb{C}\mathbb{B} = \begin{bmatrix}0 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\\ 0 & 0\end{bmatrix} = \begin{bmatrix}0 & -1\end{bmatrix}. \tag{15}$$

4. DESCRIPTOR CONTINUOUS-TIME LINEAR SYSTEMS WITH ZERO TRANSFER MATRICES

In this section the necessary and sufficient conditions for the zeroing of the transfer matrices will be established.

Theorem 3. The transfer matrix (6) of the descriptor linear continuous-time system (1) is zero matrix if and only if the following conditions are satisfied:

1. The system (1) is uncontrollable

$$\exists s \in \mathrm{C} : \operatorname{rank}\begin{bmatrix}\mathbb{E}s - \mathbb{A} & \mathbb{B}\end{bmatrix} < n \text{ or/and } \operatorname{rank}[\mathbb{E}, B] < n. \tag{16}$$

2. The system (1) is unobservable

$$\exists s \in \mathrm{C} : \operatorname{rank}\begin{bmatrix}\mathbb{E}s - \mathbb{A}\\ \mathbb{C}\end{bmatrix} < n \text{ or/and } \operatorname{rank}\begin{bmatrix}\mathbb{E}\\ \mathbb{C}\end{bmatrix} < n. \tag{17}$$

3. The product of the matrices $\mathbb{C}$, and $\mathbb{B}$ is zero matrix

$$\mathbb{C}\mathbb{B} = 0. \tag{18}$$

Proof. The proof is based on Kalman decomposition of the descriptor linear system [3]. If the system is uncotrollable and/or unobservable then in the transfer matrix (6) the cancellations of the poles and zeros occurs (for example in (14)). The transfer matrix (6) is zero matrix if and only if the condition (18) is satisfied. □

Example 2. Let us consider the system (1) with the matrices $\mathbb{E}$, $\mathbb{A}$, and $\mathbb{B}$ as given in Example 1, and the matrix as follows

$$\mathbb{C} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}. \tag{19}$$

In this case the condition (18) is satisfied since

$$\mathbb{CB} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \tag{20}$$

and the transfer matrix is zero matrix

$$\mathbf{T}(s) = \mathbb{C}[\mathbb{E}s - \mathbb{A}]^{-1}\mathbb{B} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} -1 & s-1 & -2 \\ 0 & -2 & s+1 \\ 0 & 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}. \tag{21}$$

This confirms the importance of the condition (18)

5. CONTROLLABILITY AND OBSERVABILITY OF THE DESCRIPTOR DISCRETE-TIME LINEAR SYSTEMS

Let us consider the descriptor, discrete-time, linear system

$$\mathbb{E}x_{i+1} = \mathbb{A}x_i + \mathbb{B}u_i, \quad i = 0, 1, \ldots, \tag{22a}$$

$$y_i = \mathbb{C}x_i, \tag{22b}$$

where $x_i \in \mathbb{R}^n$ is the state vector, $u_i \in \mathbb{R}^m$ is the input vector, and $y_i \in \mathbb{R}^p$ is the output vector.

It is assumed that

$$\det[\mathbb{E}z - A] \neq 0, \tag{23}$$

where $z \in \mathrm{C}$, and C is the set of complex numbers.

Definition 3. The system (22a) is called completely controllable if for any initial conditions $x_0 \in \mathbb{R}^n$ and every final state $x_f \in \mathbb{R}^n$ there exists an input $u_i \in \mathbb{R}^m$ for $i = 0, 1, \ldots, q-1$ such that $x_q = x_f$.

Theorem 4. The system (22a) is controllable if and only if

$$\operatorname{rank} \begin{bmatrix} \mathbb{E}z & \mathbb{B} \end{bmatrix} = n \quad \text{for} \quad \text{all } z \in \mathrm{C} \tag{24}$$

and

$$\operatorname{rank} \begin{bmatrix} \mathbb{E} & \mathbb{B} \end{bmatrix} = n. \tag{25}$$

Proof of this theorem is given in [3]. □

Definition 4. The system (22a), (22b) is called completely observable if there exists an integer $q > 0$ such that knowing u_i and y_i for $i = 0, 1, \ldots, q$ it is possible to find (compute) its initial state x_0.

Theorem 5. The system (22a), (22b) is observable if and only if

$$\operatorname{rank} \begin{bmatrix} \mathbb{E}z - \mathbb{A} \\ \mathbb{C} \end{bmatrix} = n \quad \text{for} \quad \text{all } z \in \mathrm{C} \tag{26}$$

and

$$\operatorname{rank} \begin{bmatrix} \mathbb{E} \\ \mathbb{C} \end{bmatrix} = n. \tag{27}$$

Proof of this theorem is given in [3]. □

The transfer matrix of the system (22) has the form

$$\mathbf{T}(z) = \mathbb{C}[\mathbb{E}z - \mathbb{A}]^{-1}\mathbb{B}. \tag{28}$$

The transfer matrix (28) represents only the controllable and observable part of the system (22) [3].

6. DESCRIPTOR, DISCRETE-TIME, LINEAR SYSTEMS WITH ZERO TRANSFER MATRICES

In this section the necessary and sufficient conditions for the zeroing of the transfer matrices will be extended to the descriptor, discrete-time linear systems.

Theorem 6. The transfer matrix (28) of the descriptor, linear system (22) is zero matrix if and only if the following conditions are satisfied:

1. The system (22) is uncontrollable

$$\exists z \in \mathrm{C}\colon \operatorname{rank} \begin{bmatrix} \mathbb{E}z - \mathbb{A} & B \end{bmatrix} < n \text{ or/and } \operatorname{rank} \begin{bmatrix} \mathbb{E} & B \end{bmatrix} < n. \tag{29}$$

2. The system (22) is unobservable

$$\exists z \in \mathrm{C}\colon \operatorname{rank} \begin{bmatrix} \mathbb{E}z - \mathbb{A} \\ \mathbb{C} \end{bmatrix} < n \quad \text{or/and} \quad \operatorname{rank} \begin{bmatrix} \mathbb{E} \\ \mathbb{C} \end{bmatrix} < n. \tag{30}$$

3. The product of the matrices $\mathbb{C}$, and $\mathbb{B}$ is zero matrix

$$\mathbb{CB} = 0. \tag{31}$$

Proof of the theorem is similar (dual) to the proof of Theorem 3. □

Example 3. Let us consider the descriptor system (22) with the matrices:

$$\mathbb{E} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbb{A} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbb{B} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbb{C} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix}. \tag{32}$$

This system (22) with matrices (32) satisfies the condition (23) since

$$\det[\mathbb{E}s-\mathbb{A}] = \begin{vmatrix} -1 & z & 2 \\ 0 & -1 & 2z \\ 0 & 0 & -1 \end{vmatrix} = -1 \neq 0. \tag{33}$$

The system with matrices (32) is uncontrollable, since it does not satisfy the condition (25)

$$\operatorname{rank}[\mathbb{E},\mathbb{B}] = \operatorname{rank}\begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 2 < n = 3. \tag{34}$$

The system is also not observable since it does not satisfy but the condition (27)

$$\operatorname{rank}\begin{bmatrix} \mathbb{E} \\ \mathbb{C} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix} = 2 < n = 3. \tag{35}$$

Note also that the condition (31) is satisfied since

$$\mathbb{CB} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{36}$$

The transfer matrix of the system with matrices (32) has the form

$$\mathbf{T}(s) = \mathbb{C}[\mathbb{E}z-\mathbb{A}]^{-1}\mathbb{B} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$
$$\times \begin{bmatrix} -1 & z & 2 \\ 0 & -1 & 2z \\ 0 & 0 & -1 \end{bmatrix}^{-1})\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{37}$$

This confirms Theorem 6.

7. CONTINUOUS-TIME LINEAR SYSTEMS WITH DELAYS IN CONTROL

Dynamical systems with different delays in state variables and/or in the control are important class of control systems (see e.g., [11], [12]). For delayed systems there exist many different kinds of controllability, e.g., relative controllability or functional controllability. In the sequel we shall concentrate on relative controllability. In this section, at first, we shall consider regular (nonsingular) linear continuous-time systems S_h with delay $h > 0$ in control, and with constant coefficients, described by the set of following equations:

$$\begin{aligned} \dot{x}(t) &= \mathbb{A}x(t) + \mathbb{B}_0 u(t) + \mathbb{B}_h u(t-h) \quad 0 < t \le t_f < \infty, \\ y(t) &= \mathbb{C}x(t), \end{aligned} \tag{38}$$

where $t \in [0,t_f]$, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $h > 0$ is a constant delay. We assume that system is regular, i.e., $\mathbb{E} = \mathbb{I}$ (identity matrix). For the above system the following relative controllability concept can be defined (see [11], and [12] for more details).

Definition 5. The system S_h is called relatively controllable on time inteval $[0,t_f]$ if for any initial relative state $x(0) \in \mathbb{R}^n$, and every finite state $x_f \in \mathbb{R}^n$ there exists an admissible input $u(t) \in \mathbb{R}^m$, for $t \in [0,t_f]$ such that $x(t_f) = t_f$.

Since the delayed system S_h is linear, and time-invariant the solution $x(t,x(0),u(t))$ exists, and can be computed using the Laplace transformation, and for $x(0) = 0$ it has the form

$$Y(s) = \mathbb{C}[s\mathbb{I}-\mathbb{A}]^{-1}(\mathbb{B}_0 + \exp(-sh)\mathbb{B}_h)U(s) = \mathbb{T}(s)U(s), \tag{39}$$

where $\mathbb{T}(s)$ is the transfer matrix for system S_h.

Remark 2. Since for $t \in [0,h]$, and $u(t) = 0$ for $t < 0$ system is behaving like the one without delays, then it can be considered similarly as the system presented in Section 2 for $\det\mathbb{E} = 0$. Therefore the transfer matrix $\mathbb{T}(s) = 0$ if matrix $\mathbb{CB}_0 = 0$.

Remark 3. However, in general for $t > h$, the above statement is not true (see e.g., [15]).

Extension of the system (38) has the following form:

$$\begin{aligned} \dot{x}(t) &= \mathbb{A}x(t) + \sum_{i=0}^{M} \mathbb{B}_i u(t-h_i) \quad 0 < t \le t_f < \infty, \\ y(t) &= \mathbb{C}x(t), \end{aligned} \tag{40}$$

where $0 \le h_i < h_{i+1} < \infty$ for $i = 0,1,2,\ldots M$ are constant delays. Quite similar remarks as above can be formulated for dynamical system with many constant delays in the control.

Now, let us consider linear, continuous-time, descriptor system with constant delay in control described by the following equations:

$$\begin{aligned} \mathbb{E}\dot{x}(t) &= \mathbb{A}x(t) + \mathbb{B}_0 u(t) + \mathbb{B}_h u(t-h) \quad 0 < t \le t_f < \infty, \\ y(t) &= \mathbb{C}x(t), \end{aligned} \tag{41}$$

where $t \in [0,t_f]$, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $h > 0$ is a constant delay. This system is a direct generalization of system (38). For delayed system (41) controllability depends on the length of time interval $[0,t_f]$. In general, it is necessery to consider two cases, namely: $t_f \le h$, and $t_f > h$.

Following [12], we obtain the following theorem.

Theorem 7. System (41) is completely, relatively controllable on interval $[0,t_f]$, where $t_f \le h$ if and only if

$$\operatorname{rank}\begin{bmatrix} \mathbb{E}s & \mathbb{B}_0 \end{bmatrix} = n \quad \text{for all} \quad s \in \mathrm{C} \tag{42}$$

and

$$\operatorname{rank}\begin{bmatrix} \mathbb{E} & \mathbb{B}_0 \end{bmatrix} = n. \tag{43}$$

Moreover, this result can be also extended to the second case, i.e. the following theorem is also true.

Theorem 8. The system (41) is relatively controllable on interval $[0,t_f]$, where $t_f > h$, if and only if

$$\operatorname{rank}\begin{bmatrix}\mathbb{E}s & \mathbb{B}_0 & \mathbb{A}\mathbb{B}_h\end{bmatrix} = n \quad \text{for all} \quad s \in \mathrm{C} \tag{44}$$

and

$$\operatorname{rank}\begin{bmatrix}\mathbb{E} & \mathbb{B}_0 & \mathbb{A}\mathbb{B}_h\end{bmatrix} = n. \tag{45}$$

Remark 4. Since for $t \in [0,h]$, and $u(t) = 0$ for $t < 0$ system (41) is also behaving like the one without delays, then it can be considered similarly as the system presented in Section 2. Therefore the transfer matrix $\mathbb{T}(s) = 0$ if matrix $\mathbb{C}\mathbb{B}_0 = 0$.

Remark 5. However, in general for $t > h$, the above statement is not true (see e.g., [15]).

8. CONCLUSIONS

In this paper controllability and observability of descriptor linear, continuous-time and discrete-time, finite dimensional systems with constant coefficients have been discussed. Using pure algebraic methods taken directly from the theory of matrices is was proved that controllability and observability of descriptor systems yields as a direct consequence the results for zeroing the transfer matrices of the systems considered. It should be pointed out that in the proofs previous results known in the literature are used.

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